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RANDOM MOTION OF SOLID PARTICLES AND ENERGY

DISSIPATION IN TWO-PHASE FLOW

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We obtain equations describing the fluctuations and energy loss of particle collisions in two-phase flow from the experimental velocity distribution functions.

The random motion of solid particles in a turbulent gas flow is one of the deciding factors in the formation of hydrodynamic structures in two-phase flow and it significantly affects the intensity of transport processes [1]. The mechanism of the generation of random motion is usually [2-5] explained in terms of the time and spatial scales of turbulence in the carrier medium and collisions between the particles and the walls of the channel.

Theoretical studies [4, 5] have shown that as the particle relaxation time increases, the degree to which the particles are drawn into the fluctuating motion of the gas decreases and approaches zero in the case when the phases slip past one another with their average velocities. This type of two-phase motion takes place in the transport of hydraulically large particles of sizes $1 \cdot 10^{-4}$ - $3 \cdot 10^{-3}$ m in chemical engineering processes such as heterogeneous catalysis, gasification of coal, adsorption, dehydration, sorting, and so on.

The mathematical description of the motion of solid particles with their collisions taken into account [6, 7] leads to equations containing the stress tensor in the solid phase and the flux vector of the fluctuating motion of the solid particles as unknowns. As shown in [6], these quantities can be expressed explicitly in terms of the macroscopic parameters of the two-phase flow in the framework of the statistical theory of dispersed systems with the help of the position and velocity distribution functions of the solid particles.

There is no information available in the literature on the parameters of random motion in two-phase flow in the presence of collisions between the solid particles. This situation has stimulated the work described in the present paper.

We carried out a systematic experimental study of the distribution functions of the longitudinal $P(u_x)$ and transverse $P(u_y)$ components of the instantaneous velocities of glass balls of diameters $d = 113 \pm 9 \mu\text{m}$ and $d = 1.18 \text{ mm}$. The measurements were done by the contact method [8], in which signals can be detected from a single collision of a solid particle against the sensitive area of the detector. With the help of an AI-256-6 pulse analyzer, we obtained data on the normal component of the amplitude of the collision impulse, the time duration of the collision, the time interval between collisions, and the flux density of particles at different points in a cross section of the channel. The study was performed on the stabilized section of the motion in a vertical pipe of diameter $D = 2R = 50 \text{ mm}$. The velocity of the medium (air) was varied between 5 and 23 m/sec. The input-output ratio of solid particles in the flow reached 25 kg·h/(kg·h). Before taking measurements in the pipe, we first calibrated the detectors by fixing the number of the channel of the pulse analyzer receiving a signal from a particle of known size, and moving with a known velocity and contacting the sensitive element of the detector at a known angle of incidence.

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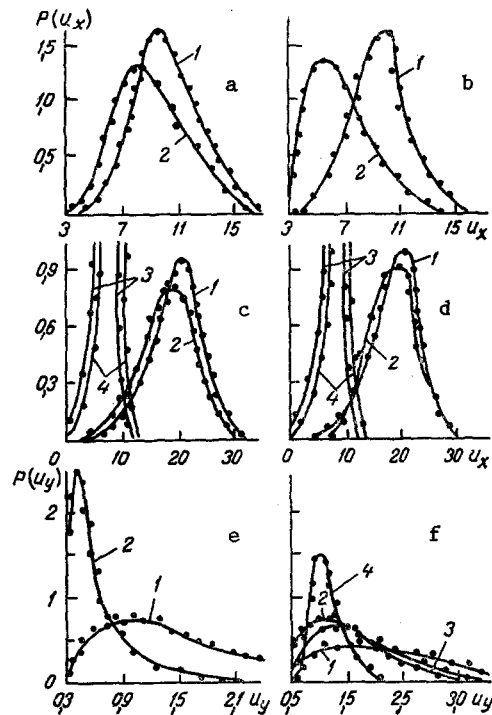


Fig. 1. Probability density curves of the longitudinal (a-d) and transverse (e, f) components of the instantaneous velocity of glass balls of diameter $d = 113 \mu\text{m}$ (curves 1 and 2) and 1.18 mm (curves 3 and 4): (a), (c), center of the channel $R = 0$; (b), (d), $R = 24 \text{ mm}$; (a), (b), $w = 12.3 \text{ m/sec}$, 1) $\mu_p = 0.809 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 2) 19.47 ; (c), (e), 1, 2) $w = 23.1 \text{ m/sec}$, 1) $\mu_p = 0.48 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 2) 7.1 ; 3, 4) $w = 23.5 \text{ m/sec}$, 3) $\mu_p = 0.48 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 4) 8.6 ; (d), $w = 12.3 \text{ m/sec}$, 1) $\mu_p = 0.24 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 2) 21.65 ; (f), 1, 2) $w = 23.1 \text{ m/sec}$, 1) $\mu_p = 0.85 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 2) 10.44 ; 3, 4) $w = 23.5 \text{ m/sec}$, 3) $\mu_p = 1.0 \text{ kg}\cdot\text{h}/(\text{kg}\cdot\text{h})$, 4) 16.5 .

The nature and evolution of the measured probability densities of the longitudinal $P(u_x)$ and transverse $P(u_y)$ components of the instantaneous velocities of the particles are shown in Fig. 1.

Note the slight deviation of the curves $P(u_x)$ from a normal distribution for $\mu_p \leq 5$ independently of the velocity of the medium and the size of the particles. The average static distribution of particles in space is qualitatively uniform and does not depend on the radial coordinate, at least to within several particle diameters of the walls of the pipe. Note the large widths of the velocity distribution functions. In particular, the limiting velocity of steady motion of an isolated particle in free space $u_x = w - v$ is included within the widths of the $P(u_x)$ curves in the case of particles of diameter 1.18 mm (Figs. 1c and 1d; curves 3 and 4). The functions $P(u_x)$ and $P(u_y)$ for these particles were analyzed in detail earlier [9]. The velocities of a significant (up to 15%) fraction of the particles of diameter $113 \mu\text{m}$ are larger than the average velocity of the medium. The presence of "fast" particles in the flow was noted earlier in [10, 11] in a study of two-phase flows using high-speed motion pictures. It is evidently a consequence of the change in the structure of the turbulence of the medium induced by the solid particles and the participation of the particles in the large-scale fluctuating motion of the gas. The data on the distribution of the transverse velocities of the particles u_y (Figs. 1e and 1f) were obtained on the channel wall and describe the motion of the particles from the axis of the flow field to its periphery. The shape of the function $P(u_y)$ is practically independent of the particle diameter. An increase in particle concentration and velocity of the medium is accompanied by a steady decrease in the width of the distribution $P(u_y)$ and a shift of the most probable value of the velocity toward smaller u_y .

A physical interpretation of this result is that the two-phase flow becomes more ordered as μp and w increase, i.e. the direction of the velocity vectors of the majority of the particles are oriented along the flow axis.

The distributions $P(u_x)$ and $P(u_y)$ can be used to compute the average velocity components of the particles

$$\bar{u}_i = \int u_i P(u_i) du_i, \quad (1)$$

the mean-square values of the velocity components

$$\overline{u_i^2} = \int (u_i - \bar{u}_i)^2 P(u_i) du_i, \quad (2)$$

the average relative velocity of two colliding particles (subscripts 1 and 2)

$$\bar{c} = \iint (u_1 - u_2) P(u_1) P(u_2) du_1 du_2. \quad (3)$$

The collision frequency of a single particle moving in a flow of particles with number density $n = 6\beta/\pi d^3$ is

$$f_0 = \pi d^2 n \bar{c}. \quad (4)$$

In the absence of explicit forms of the distribution functions $P(u_i)$, we calculated the relative velocity \bar{c} using as a first approximation the following well-known relation from the kinetic theory of a nonuniform gas [12]

$$\bar{c} = \sqrt{2\bar{u}'}, \quad (5)$$

where $\bar{u}' = \sqrt{\overline{u_x'^2} + 2\overline{u_y'^2}}$. We then obtain the following expression for the collision frequency of a single particle

$$f_0 = \sqrt{2} \pi d^2 n \bar{u}'. \quad (6)$$

The rms velocity components (velocity fluctuations) of the particles $\overline{u_i'}$ calculated from (2) are shown in Fig. 2. Also, in the usual terminology of turbulent motion the intensities of random motion of the solid particles in the x and y directions are also shown: $Tu_x = \overline{u_x'^2}/\overline{u_x}$ and $Tu_y = \overline{u_y'^2}/\overline{u_y}$.

Because of the ordered motion of the two-phase flow, significant anisotropy in the fluctuation transport of particles is observed, i.e., $\overline{u_x'^2} > \overline{u_y'^2}$. The intensities of random motion of the particles are of the same order of magnitude and reach values typical of the intensities in turbulent gas flow [13]. The contrast between the functions $\overline{u_x'^2}/\overline{u_x} = f(\mu p)$ for particles of diameter 0.113 mm and 1.18 mm is due to the different values of the particle collision factor B (Fig. 3). The quantity B is the ratio of the collision time of a single particle in the flow to the time between its successive collisions with the walls of the channel, $B = t_u/t_w$. For the larger particles, the quantity B goes from greater than unity to less than unity with an increase in their concentration in the flow. At high concentrations the effect of screening of the particles in the flow becomes important, and the collision frequency and momentum loss at the walls decreases and hence the mean velocity increases. From Fig. 3a we have $B < 1$ on the entire μp axis for particles of diameter $d = 113 \mu m$. Also because the average velocity of the small particles in the flow is close to the equilibrium velocity of an isolated particle $u_x = w - v$, the effect of screening is weak and collisions with the walls lead to a decrease in the average velocity of the particles and hence to an increase in $\overline{u_x'^2}/\overline{u_x}$.

Certain general features of the behavior of solid particles in a turbulent gas flow can be understood in terms of the particle relaxation time

$$\tau_r = \frac{4\rho_B d}{3\rho_C D(w-u)}. \quad (7)$$

The ratio of the collision time t_u to τ_r (Fig. 3b) in the region studied here is described by a universal function of the input-output ratio of the particles:

$$\frac{t_u}{\tau_r} = \frac{A}{\mu_P} \quad (8)$$

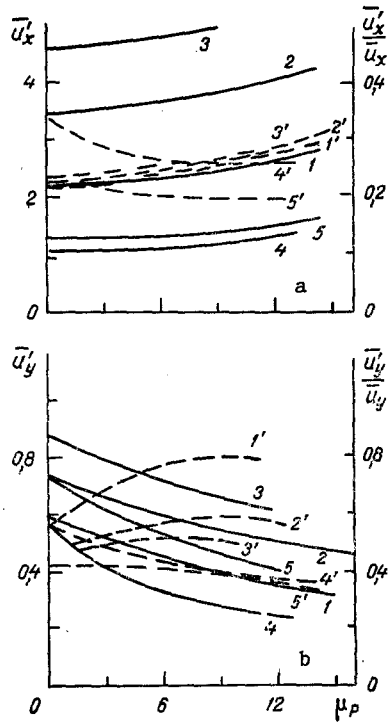


Fig. 2

Fig. 2. Longitudinal (a) and transverse (b) components of the rms velocity fluctuation (curves 1-5) and intensity of the random motion (curves 1'-5') of solid particles: 1-3, 1'-3') $d = 113 \mu\text{m}$; 4, 5; 4', 5') $d = 1.18 \text{ mm}$; 1, 1') $w = 123 \text{ m/sec}$; 2, 2') 17.4 ; 3, 3') 23.1 ; 4, 4') 16.5 ; 5, 5') $w = 23.5 \text{ m/sec}$.

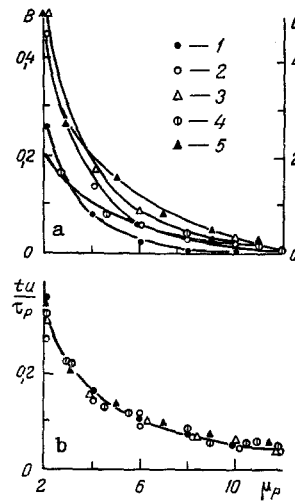


Fig. 3

Fig. 3. Dependence of the characteristic times of particle motion on the input-output ratio of particles: a) collision factor of the particles, b) ratio of the collision time to the relaxation time: 1-3) $d = 113 \mu\text{m}$, 4, 5) $d = 1.18 \text{ mm}$; 1) $w = 12.3 \text{ m/sec}$; 2) 17.4 ; 3) 23.1 ; 4) 16.5 ; 5) $w = 23.5 \text{ m/sec}$.

with the proportionality constant $A = 0.625$.

On the other hand, we have from (6) and (7)

$$\frac{t_u}{\tau_r} = \frac{c_D(w - \bar{u})\bar{u}}{8\sqrt{2}\bar{u}'w\mu_p} \quad (9)$$

Solving (8) and (9) simultaneously, we obtain an expression for the rms velocity fluctuation of the particles

$$\bar{u}' = \frac{c_D(w - \bar{u})\bar{u}}{8\sqrt{2}wA} \quad (10)$$

from which we obtain the important conclusion that the random motion of solid particles in the two-phase flow is determined by streamline flow of the medium around the particles.

Comparison of the values of \bar{u}' calculated from (10) with the experimental data (Fig. 2) gives a correction factor of 4.8. The introduction of such a correction factor is physically admissible because the assumption of a Maxwellian velocity distribution of the particles in (6) is just a first approximation to the experimental distribution functions shown in Fig. 1. In addition, a strong assumption is the linear dependence between the forces acting on a particle and the relative velocity $(w - \bar{u})$ used to calculate the relaxation time in (7). Taking into account the correction factor of 4.8, the rms velocity fluctuation (10) of a particle in two-phase flow in the Reynolds number region $\text{Re} = 20-1000$ corresponding to streamline flow of the medium takes the form:

$$\bar{u}' = 0.68c_D(w - \bar{u}) \frac{\bar{u}}{w}. \quad (11)$$

This equation describes the experimental data to within $\pm 13\%$. The large value of the intensity of the random motion of the particles u'/\bar{u} (Fig. 2) characterizes the two-phase flow considered here as a nonequilibrium system [1]. Because of relaxation processes (such as energy dissipation in inelastic collisions of the solid particles) the system approaches an equilibrium state and the random motion of the particles should decrease. It follows from the particle velocity distribution functions (see Fig. 1) that this is an inherent property of two-phase flow. If energy is supplied to the medium, a steady state can be established in the system, in which the quantity u'/\bar{u} takes on a constant value. Relaxation processes in the flow still exist, but they are compensated by the supply of energy to the gas [7, 14].

We consider the quasi-stabilized section of vertical flow of a suspension in gas. The instantaneous value of the kinetic energy of the solid particles is written as a sum of an average kinetic energy and a fluctuation:

$$E = \bar{E} + E', \quad (12)$$

where the two terms in this expression characterize the motion of the system of particles as a whole with the center-of-mass velocity and the random motion of the particles about the center of mass, respectively. For inelastic binary collisions, a part ΔE_{12} of the kinetic energy

$$\Delta E_{12} = \frac{m}{4} \bar{c}_{xy}^2 (1 - k^2) \quad (13)$$

is dissipated [7] and then regenerated by irreversible energy losses in the gas phase.

The quantity \bar{c}_{xy} in (13) is the component of the relative velocity of the two particles along the line of the collision, which in the case of spherical particles coincides with the line joining the centers of the two spheres. Following [15], we assume that all directions of the line joining the centers of the two particles are equally probable and the average value of the angle between this line and a coordinate axis is $\pi/4$. Then

$$\bar{c}_{xy} = \bar{c} \cos \frac{\pi}{4}. \quad (14)$$

Assuming the number of collisions N per unit time is [12]

$$N = \frac{1}{2} \sqrt{2} n^2 \pi d^2 \bar{u}', \quad (15)$$

we obtain from (13) and (14) an equation for the total energy dissipation from particle collisions inside a volume of the pipe $\pi R^2 L$

$$\Delta E = \frac{3 \sqrt{2} \beta^2 \bar{u}'^3 \pi R^2 L (1 - k^2) \rho_s}{4d}. \quad (16)$$

The pressure loss from particle-particle collisions in two-phase flow can be obtained from the energy dissipation using the idea of continuous generation of energy of the random motion of solid particles discussed above. According to this mechanism, the work done by the dissipative forces in an isolated volume during a time Δt is compensated by the work done by the gas flowing a certain path length in the same time interval:

$$\Delta E \Delta t = \Delta P_u \pi R^2 w \Delta t, \quad (17)$$

hence with the use of (16) we obtain

$$\Delta P_u = \frac{1.05 \beta^2 \bar{u}'^3 (1 - k^2) \rho_s L}{dw}. \quad (18)$$

Calculation of ΔP_u from (18) with $k = 0.94$ [16] for the coefficient of restitution of glass particles and with the use of the experimental data of the present paper (Fig. 4) shows the importance of the contribution of particle-particle collisions to the total energy loss in a two-phase flow.

To a large extent this assertion is correct for the flow of a suspension of small particles in a gas. For example, for particles of diameter $d \sim 0.1$ mm, the quantity ΔP_u increases with increasing μp and w and reaches $(0.3-0.4)\Delta P$. An increase in the size of the particles leads to a decrease in the component ΔP_u . According to the data of Fig. 4, the contribution of ΔP_u to the total pressure loss in a flow with particles $d \sim 1$ mm is quite small (3-5%). Comparison of ΔP_u with losses due to the suspension of solid particles ΔP_s ,

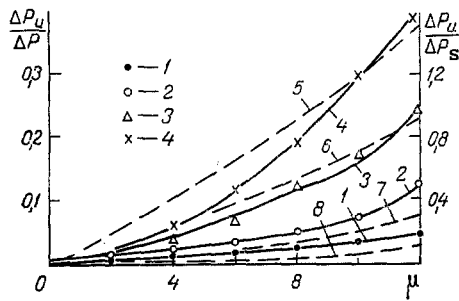


Fig. 4. Contribution of particle-particle collisions to the energy loss in two-phase flow: 1-4) $\Delta P_u / \Delta P$; 5-8) $\Delta P_u / \Delta P_s$; 1, 8) $w = 23.5$ m/sec; 2, 7) 12.3; 3, 6) 17.4; 4, 5) 23.1 m/sec; 1, 8) $d = 1.18$ mm; 2-7) 113 μ m.

which is one of the principal components of the total energy lost (see Fig. 4) indicates that in the region of the parameters μ , w , d important in practice, the quantities ΔP_u and ΔP_s are of the same order.

Because of the lack of information on the random motion of particles, the component ΔP_u of the loss has generally been determined [17-19] by resorting to the conventional picture of energy loss in the flow of continuous medium based on the drag coefficient. At best, such an empirical approach leads [17-19] to estimates of the total energy loss in two-phase flow. The course taken in the present paper leads (according to (11) and (18)) to a physically valid description of the separate components of the energy loss using only data on the average velocities and the physical and mechanical properties of the particles.

NOTATION

u , \bar{u}' , c , average velocity, rms velocity fluctuation, and relative velocity of the particles; v , terminal velocity; w , average velocity of the gas; μ , β , n , input-output mass, volumetric, and number ratios of the particles; f_0 , collision frequency of a single particle; t_u , time between two successive particle-particle collisions; t_w , time between two successive particle-wall collisions; τ_r , relaxation time; ρ , ρ_s , densities of the gas and the solid particles; c_D , drag coefficient; k , coefficient of restitution; ΔP , total pressure loss of the gas; ΔP_u , pressure loss of the gas resulting from particle collisions in two-phase flow; ΔP_s , pressure loss of the gas due to suspension of the material.

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ANALYSIS OF MELTING IN A VERTICAL ANNULAR GAP IN
 CONDITIONS OF HEAT TRANSFER WITH AN INDUCED HEAT-
 CARRIER FLUX

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Experimental results on the melting of materials in a vertical annular gap in conditions of heat transfer with an induced heat-carrier flux are obtained and generalized.

There has been extensive experimental and theoretical investigation of the heat transfer on melting (solidification) [1-8], paying particular attention to the influence of natural convection on the heat-transfer intensity, the determination of the phase-interface position, and the derivation of generalizing relations for the volume fraction of melt with boundary conditions of types I and II. In practice, melting (solidification) with heat transfer to an induced heat-carrier flux is of great interest. The solution of such problems is particularly urgent in creating effective and economical heat storage units operating together with various types of power equipment. At present, there is limited information available here; it basically pertains to the theoretical investigation of heat transfer in the exterior solidification of plane or cylindrical channels cooled by a heat-carrier flux [9-11]. Note that the solutions obtained take no account of the role of natural convection in the melt, whereas convection has a significant influence on the heat-transfer characteristics in melting (solidification). In addition, there are no formulas as yet for the calculation of the integral heat-transfer characteristics on melting (solidification) in conditions of induced heat-carrier flow.

The aim of the present work is to elucidate the basic laws of heat transfer on melting in a vertical annular gap in conditions of induced heat-carrier flow, and to obtain a generalizing dependence for the calculation of the stored energy.

Experimental investigation of the heat transfer on melting is undertaken on five models. Each model is a system of two vertical coaxial cylindrical tubes: a working channel and the external shell of the model. There is induced motion of the heat carrier along the working channel, and the annular gap is filled with melting material. One model is briefly described here. The working channel of the model is made from a stainless steel tube of internal diameter 2.8 mm and wall thickness 0.1 mm, with a section of hydrodynamic stabilization. At the working-chamber inlet and outlet, there are mixing chambers for measuring the heat-carrier temperature. The external shell of the model is also made from a stainless steel tube (internal diameter 25 mm; wall thickness 0.4 mm). Chromel-Copel thermocouples are welded to the outer surface of the working channel and the external shell of the model, so as to record the wall temperature distribution over the height of the model in the course of the experiments.

One basic feature of the model is the heat-insulating system, which is required to ensure adiabatic conditions at the external shell. It consists of a vacuum chamber, ohmic heaters, and a heat-insulating coating. Ohmic heaters are placed on the outer surface of

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